Homework 5
EL-GY 9123 – Game Theory for Multi-Agent Systems
Fall 2014, New York University
Due Date: Dec. 12, 2014

**Reading Assignment 1.** Read [GO]: Chps IX, X (pp. 211-224), XI, XII (pp. 261-303), and XIII (pp. 313-334).

**Reading Assignment 2.** Read [FT]: Chapter 8.

**Reading Assignment 3.** Read the following materials:

1. L. Shapley’s Nobel Prize Lecture


Problem 1. Consider a third-price auction, i.e., an object is given to the highest bidder, but the bidder pays an amount equal to the third-highest bid. Assume that the valuations of the $N$ bidders are i.i.d. Let $f_\Theta$ and $F_\Theta(\theta)$ be the pdf and cdf, respectively, of each bidder's valuation. Assume that $\ln F_\Theta$ is a concave function. Show that the bidding strategy

$$\mu(\theta_i) = \theta_i + \frac{F_\Theta(\theta_i)}{(N-2)f_\Theta(\theta_i)}, \forall i.$$ 

is a symmetric BNE for this problem. Note: Assume differentiability and sufficiency of the first-order necessary conditions as and when required.

Problem 2. Suppose that there is a single buyer and a single seller for some object. The buyer has a valuation $\theta$ which is her private information. The seller knows that that $\theta$ is drawn from a pdf $f_\Theta$. Suppose that the seller sets a price $p$ at which he is willing to sell. Show that the optimal $p$ is $\Psi^{-1}(0)$, where $\Psi$ is the virtual valuation in Myerson's mechanism. Note: You can just use the first-order necessary condition only to determine the optimal solution.

Problem 3. Exercise 4.5 of [FT].

Problem 4. Consider a two-person game with $(u, v)$ denoting the utility levels of the two players. Assume that this pair is restricted to an ellipsoidal region defined by $(u - 1)^2 + 4v^2 \leq 4$. Find the Nash bargaining solution to this game when

(i) The conflict payoff is $(0, 0)$.

(ii) The conflict payoff is $(1, 0)$.

Problem 5. Read section X.5 of Owen, and solve Problem 4 on page 233.

Problem 6. Consider the 3-person cooperative game with the characteristic function: $v(\emptyset) = v(\{2, 3\}) = v(\{1, 3\}) = v(\{1, 2\}) = 0, v(\{1\}) = v(\{2\}) = v(\{3\}) = 2, v(\{1, 2, 3\}) = 3$

(i) Is it a constant-sum game?

(ii) Is it a superadditive game?

(iii) Show that the core of this game, $C(v)$, is empty.

Problem 7. Show that the 3-person game with the characteristic function $v(\emptyset) = v(\{2, 3\}) = v(\{1, 3\}) = v(\{1, 2\}) = 0, v(\{1\}) = v(\{2\}) = v(\{3\}) = 1, v(\{1, 2, 3\}) = 3$ has a nonempty core, and obtain it.

Problem 8. Problem 1 from the text Owen, page 311.

Problem 9. Evaluate the Shapley vector for the four-person game defined by the characteristic function:

$$v(\emptyset) = 0, v(\{i\}) = 0, i = 1, 2, 3, 4, v(\{2, 3\}) = v(\{3, 4\}) = v(\{2, 4\}) = 0, v(\{1, 2, 3, 4\}) = 1$$

and $v(\{i, j\}) = v(\{i, j, k\}) = 1$ for all other two- or three-person coalitions.
Problem 10. Read the following paper


and find the $H_\infty$ filter for the following system

$$x_{k+1} = \begin{pmatrix} 0.5079 & 0.7594 \\ -0.7594 & 0.2801 \end{pmatrix} x_k + \begin{pmatrix} 0.4921 \\ 0.7594 \end{pmatrix} w_k,$$

$$y_k = \begin{pmatrix} 0 & 1 \end{pmatrix} x_k + v_k,$$

$$z_k = \begin{pmatrix} 1 & 0 \end{pmatrix} x_k,$$

where $x_k \in \mathbb{R}^2$, $z_k \in \mathbb{R}^2$, $y_k \in \mathbb{R}$, and $v_k, w_k \in \mathbb{R}$ are process and measurement noise respectively. The initial condition is $x_0$.

- State the $H_\infty$ filter to obtain the state estimate $\hat{x}_k$ and the measurement estimate $\hat{z}_k$.

- State the Kalman filter (i.e., $\gamma_K = 0$) to obtain the state estimate $\hat{x}_k$ and the measurement estimate $\hat{z}_k$.

- Find the gain of the Kalman filter and $H_\infty$ filter with $\gamma_H = 1.24$.

- Use MATLAB simulations to compare the performance of two filters.